Hong-Kong, Jan 14th 2020

EXPERIMENTAL ENTANGLEMENT OF TEMPORAL ORDERS

G. Rubino, L. A. Rozema, F. Massa, M. Araújo, M. Zych, C. Brukner and P. Walther

Illustration from P. Ball, *Nature* **546**, 590 (2017)

Typically, in all the well-established physical theories, it is assumed that the order between events is pre-defined.

However, it was recently realised that quantum theory may predict the existence of processes exhibiting a ***genuinely indefinite causal structure***.

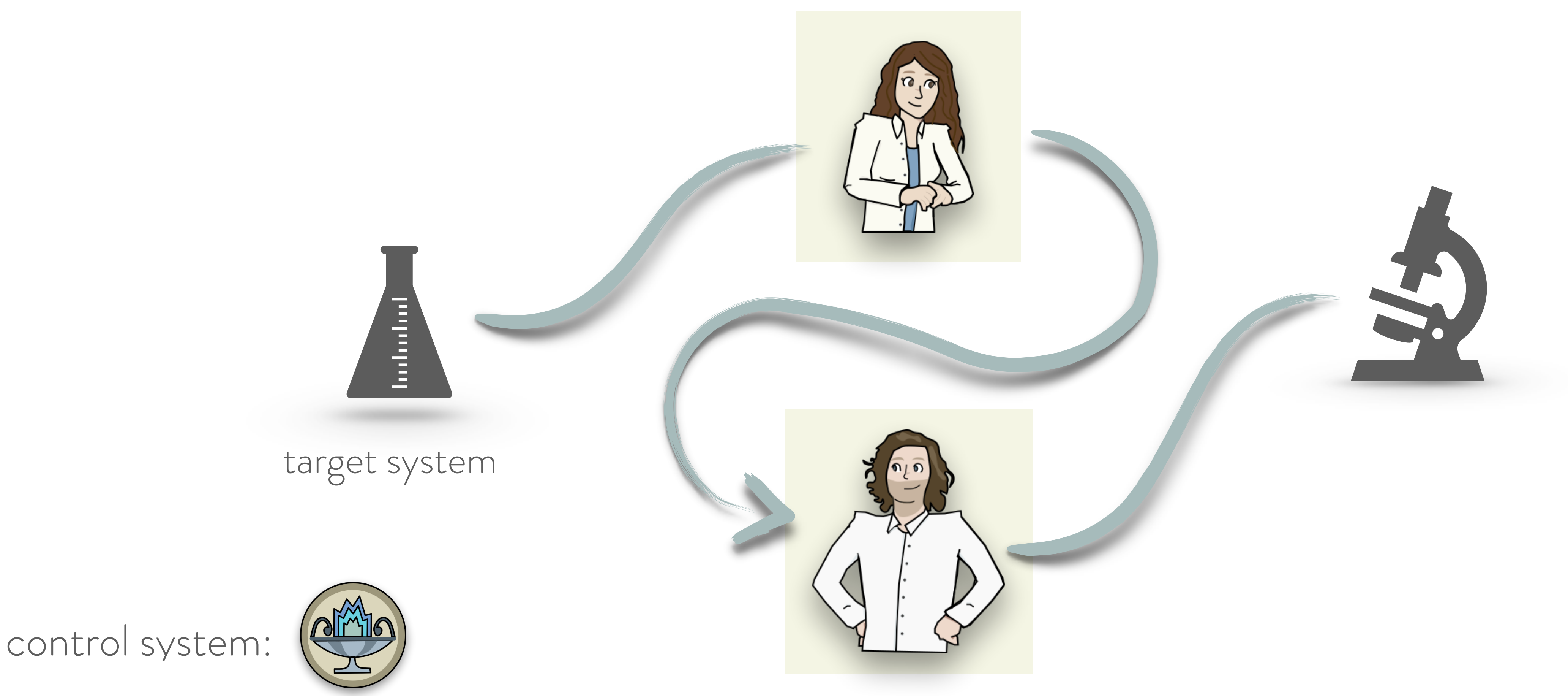
[1] L. Hardy, Towards quantum gravity: a framework for probabilistic theories with non-fixed causal structure, J. Phys. A: Math. Theor. ***40****, 3081 (2007).*

[2] G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron, Quantum computations without definite causal structure, Phys. Rev. A 88, 022318 (2013).

[3] O. Oreshkov, F. Costa, C. Brukner, Quantum correlations with no causal order, Nat. Commun. ***3****, 1092 (2012).*

genuinely indefinite causal structure:

a quantum process is called causally separable if it can be decomposed as a convex combination of causally ordered processes, otherwise it is causally non-separable.



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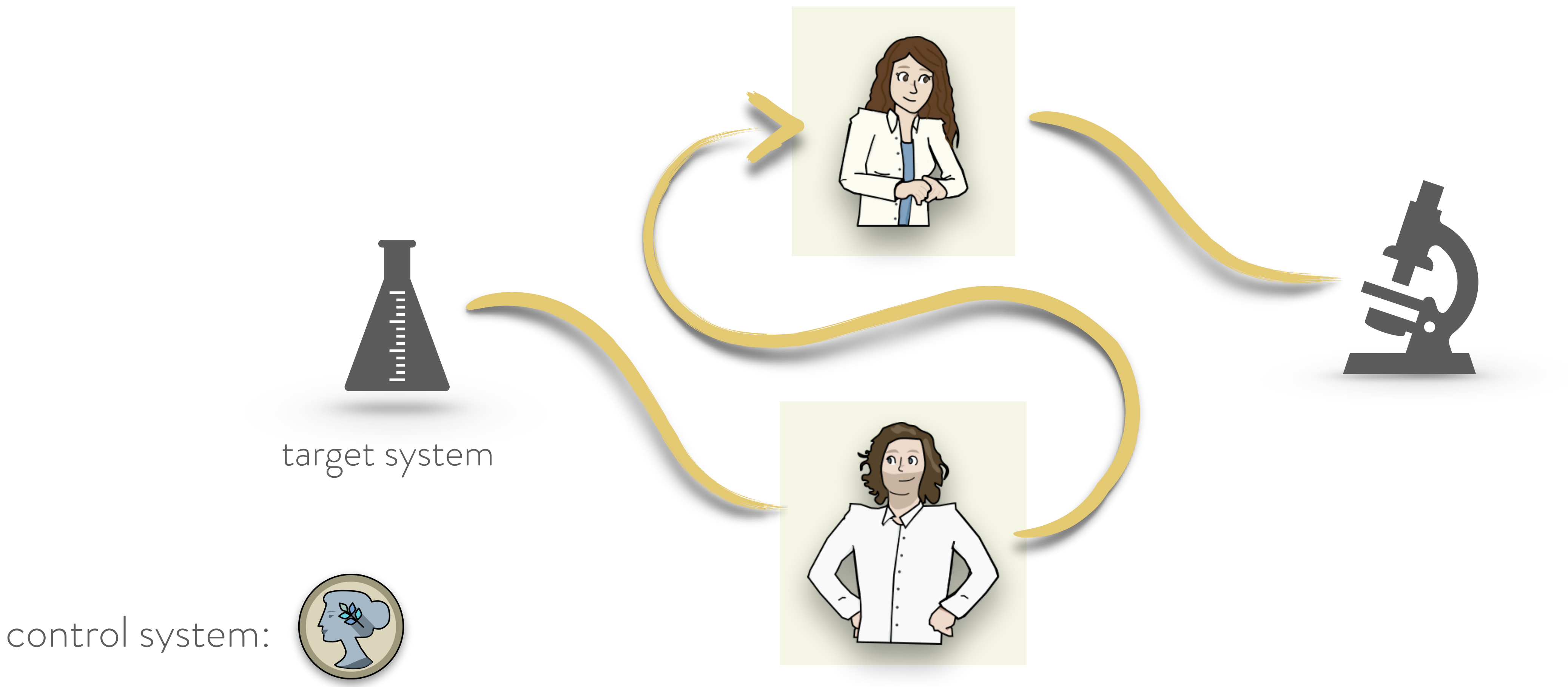

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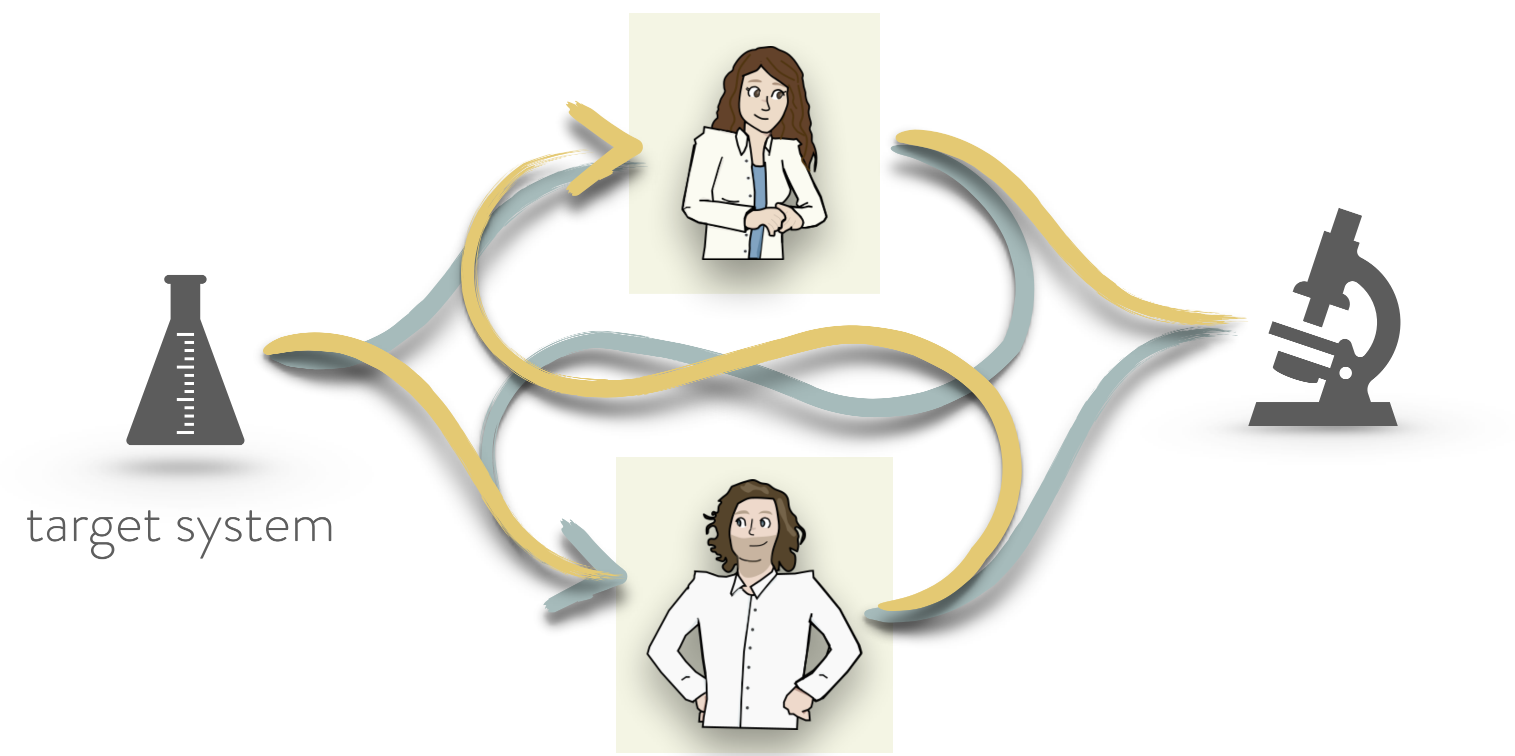


control system: $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$$\cancel{\Omega(\psi) = \zeta \cdot \mathcal{B}(\mathcal{A}(\psi)) + (1-\zeta) \cdot \mathcal{A}(\mathcal{B}(\psi))}$$

genuinely indefinite causal structure:

a quantum process is called causally separable if it can be decomposed as a convex combination of causally ordered processes, otherwise it is causally non-separable.

Causal Witness:

device-dependent way of certifying indefinite causal orders by measuring properly-designed quantum observables.

[4] G. Rubino, et al., Experimental Verification of an Indefinite Causal Order, Science Advances 3 (2017).
[5] K. Goswami, et al., Indefinite Causal Order in a Quantum Switch, Phys. Rev. Lett. 121, 090503 (2018).

Oreshkov, Costa and Brukner proposed device-independent ways of certifying indefinite causal orders via 'causal inequalities'.

Any probabilities that show signalling in only one direction, or that is a convex mixture of those which allow signalling only in one direction satisfy causal inequalities.

It can be shown that the quantum SWITCH satisfies all such causal inequalities, and, currently, it is not known how to realise a process which violates a causal inequality.

[3] O. Oreshkov, F. Costa, C. Brukner, Quantum correlations with no causal order, Nat. Commun. 3, 1092 (2012).

The question behind this work is then ***if it is at all possible to prove the existence of an indefinite causal order in a manner that applies to a broader class of theories, not only to quantum theory.***

OVERVIEW

Within this talk, I will provide an affirmative answer to this question by experimentally violating a Bell inequality applied to temporal orders.

A no-go theorem for temporal orders

Experimental realisation and results

Outlook and perspectives

A NO-GO THEOREM FOR TEMPORAL ORDERS

The no-go theorem which I present here was previously derived in the context of gravity [6].

Consider ω to be a state of the composite system of the control system (governing the order in which the operations are applied), and the target system (on which the operations are performed).

[6] M. Zych, F. Costa, I. Pikovski, C. Brukner, Bell's Theorem for Temporal Orders, Nature Communications 10, 3772 (2019).

A NO-GO THEOREM FOR TEMPORAL ORDERS

1. *The initial joint state of the target system is local (i.e., it does not violate a Bell inequality).*

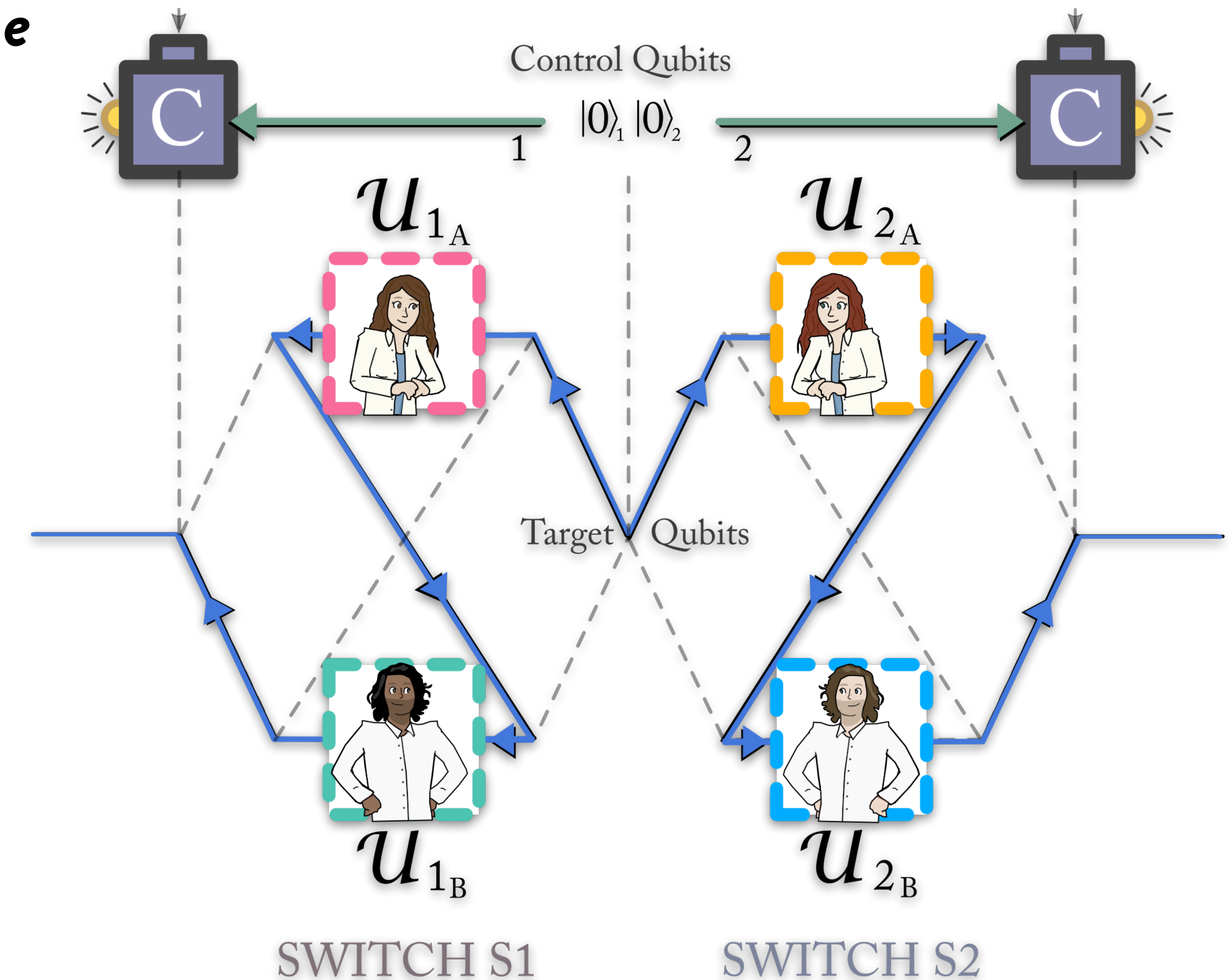
2. *The laboratory operations are local transformations of the target systems (i.e., they do not increase the amount of a violation of Bell inequalities between the two target systems).*

3. *The order of local operations on the two target systems is well-defined.*

Theorem. *No states, set of transformations and measurements which obey assumptions **1.-3.** can result in violation of Bell's inequalities.*

A NO-GO THEOREM FOR TEMPORAL ORDERS

Sketch of the Scheme:



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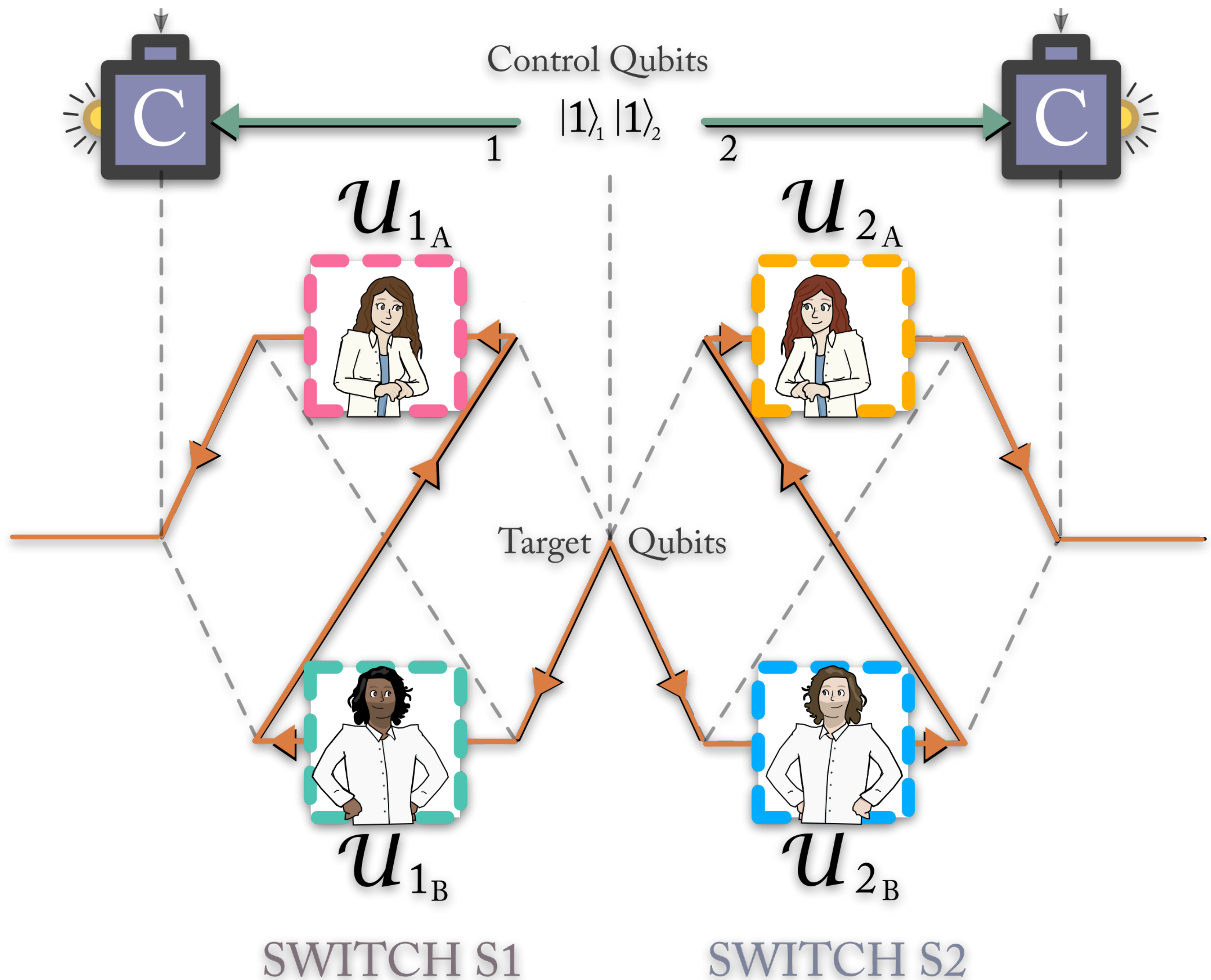

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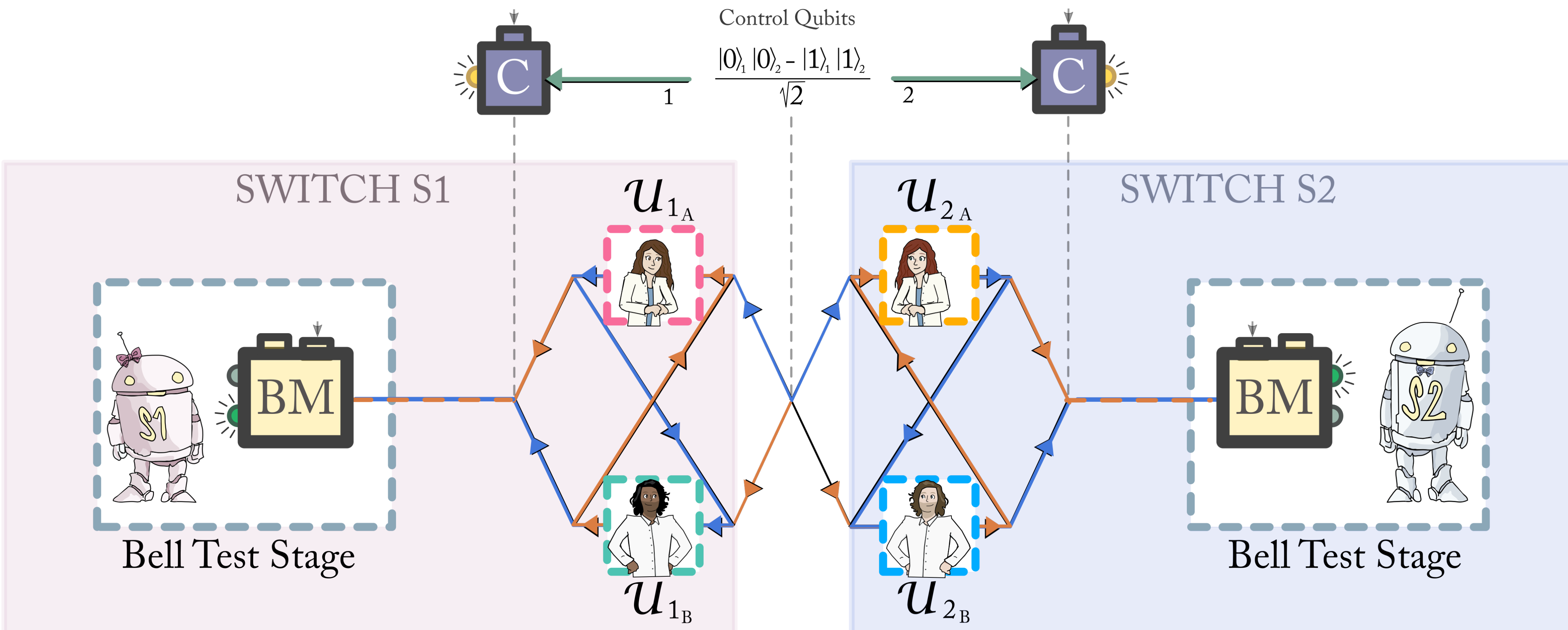
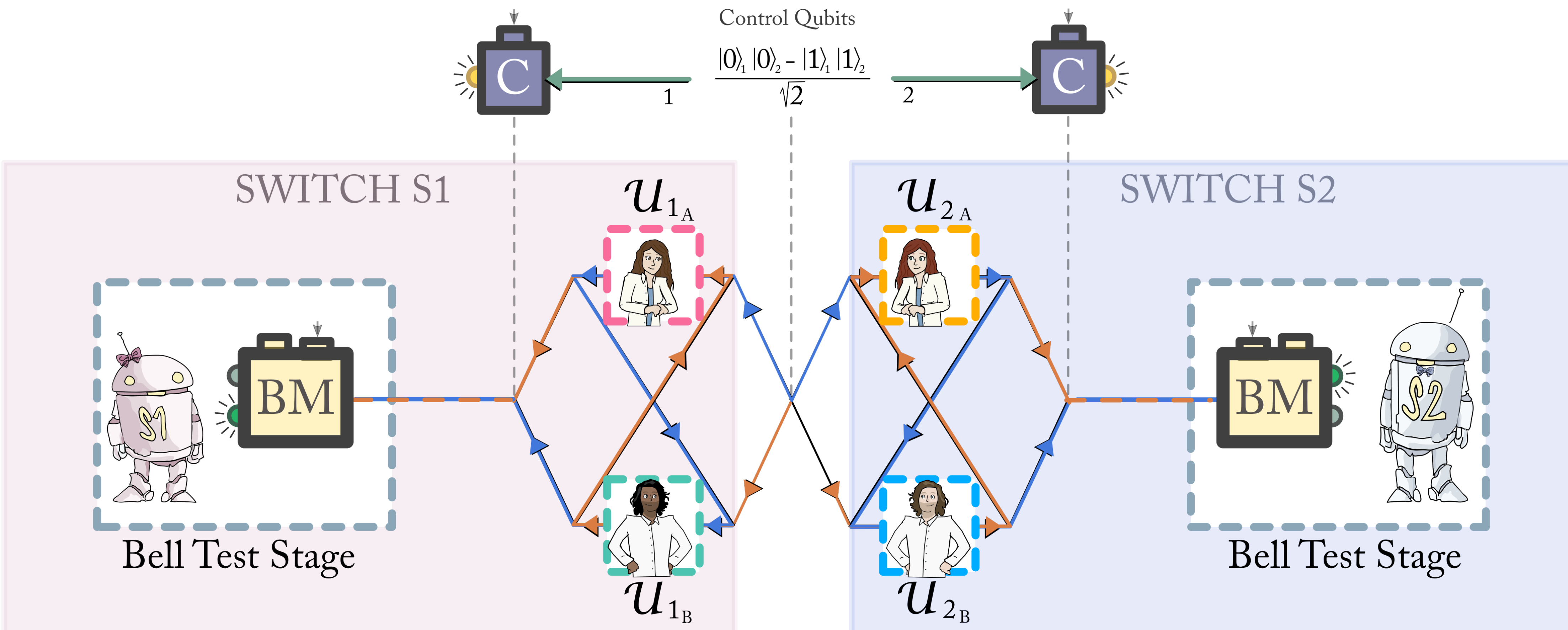


1) Initial state: $|0\rangle_1^T \otimes |0\rangle_2^T \otimes \left(\frac{|0\rangle_1^C \otimes |0\rangle_2^C - |1\rangle_1^C \otimes |1\rangle_2^C}{\sqrt{2}} \right)$

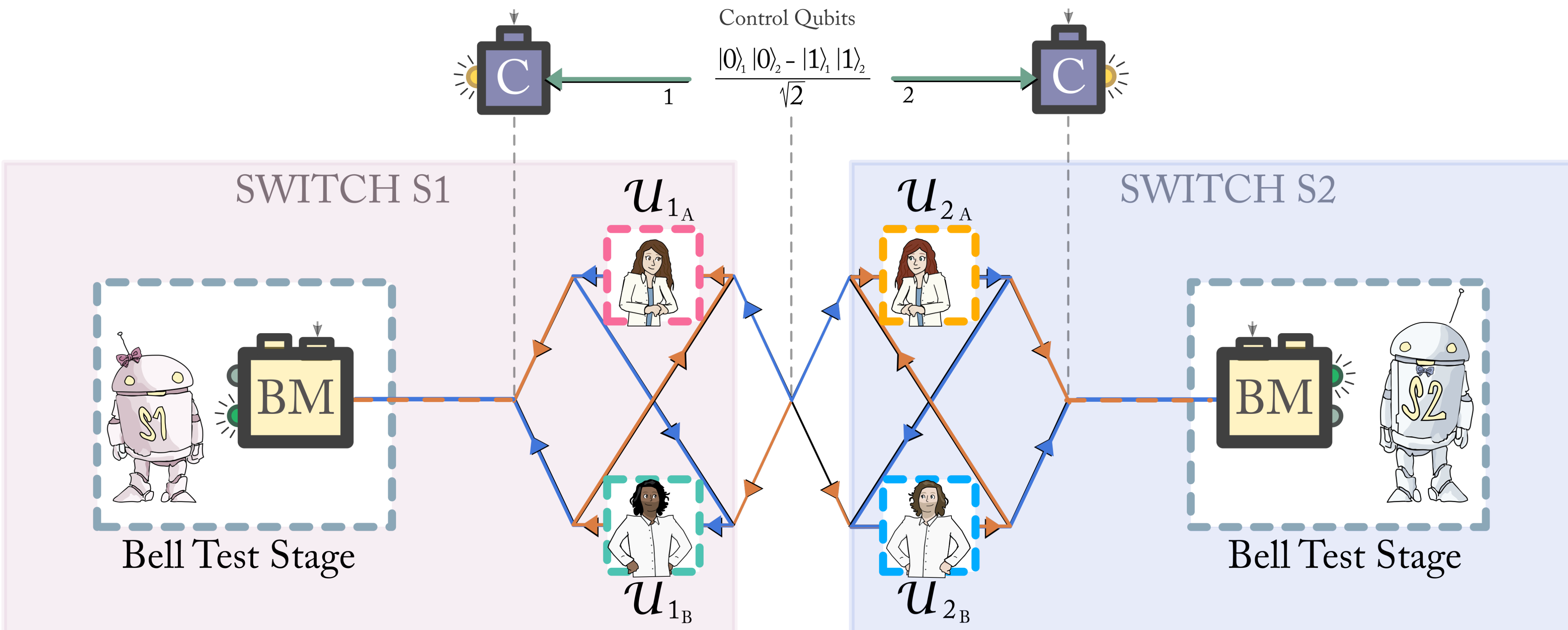
A NO-GO THEOREM FOR TEMPORAL ORDERS



2) Application of the gates:

$$\frac{1}{\sqrt{2}}\left(\mathcal{U}_{1_B}\circ\mathcal{U}_{1_A}|0\rangle_1^T\right)\otimes|0\rangle_1^C\otimes\left(\mathcal{U}_{2_B}\circ\mathcal{U}_{2_A}|0\rangle_2^T\right)\otimes|0\rangle_2^C$$
$$-\frac{1}{\sqrt{2}}\left(\mathcal{U}_{1_A}\circ\mathcal{U}_{1_B}|0\rangle_1^T\right)\otimes|1\rangle_1^C\otimes\left(\mathcal{U}_{2_A}\circ\mathcal{U}_{2_B}|0\rangle_2^T\right)\otimes|1\rangle_2^C$$

A NO-GO THEOREM FOR TEMPORAL ORDERS



3) Projection of the control qubit on $|+\rangle^C, |-\rangle^C$

$$\frac{1}{\sqrt{2}}(\mathcal{U}_{1_B} \circ \mathcal{U}_{1_A}|0\rangle_1^T \otimes \mathcal{U}_{2_B} \circ \mathcal{U}_{2_A}|0\rangle_2^T - \mathcal{U}_{1_A} \circ \mathcal{U}_{1_B}|0\rangle_1^T \otimes \mathcal{U}_{2_A} \circ \mathcal{U}_{2_B}|0\rangle_2^T)$$

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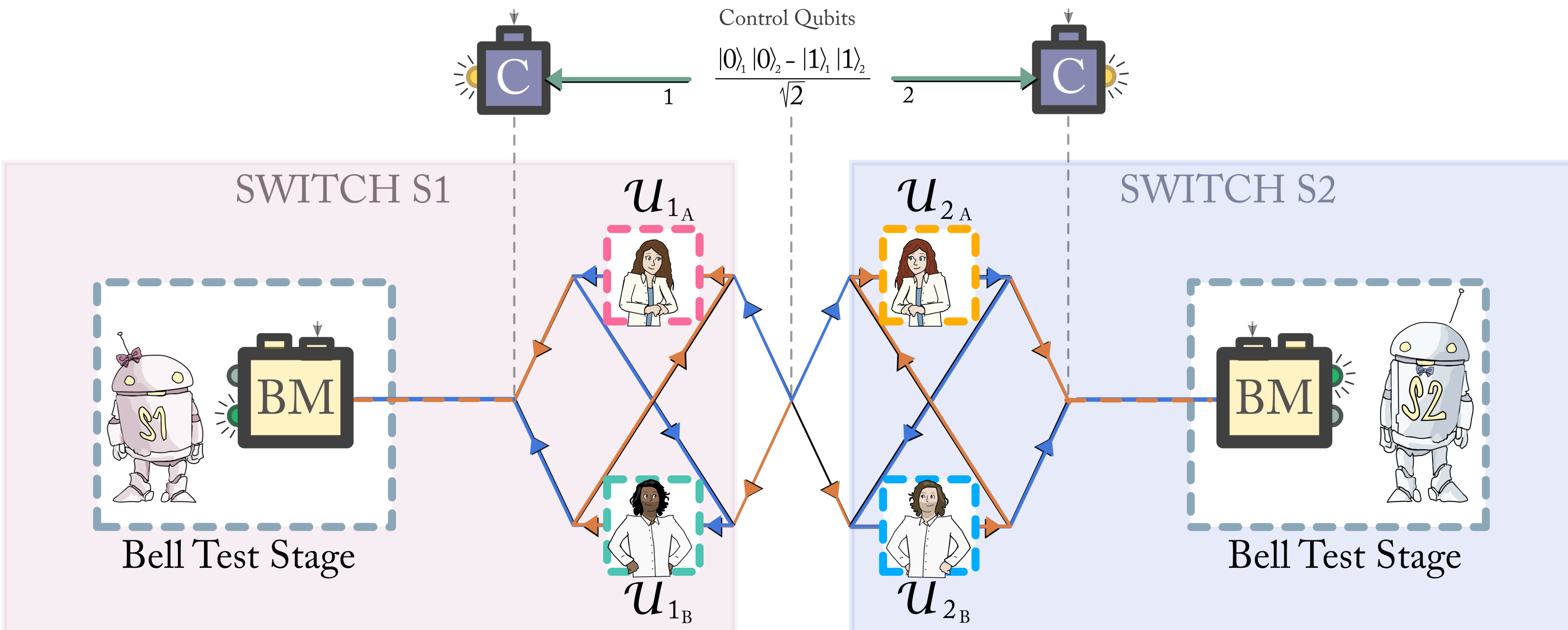
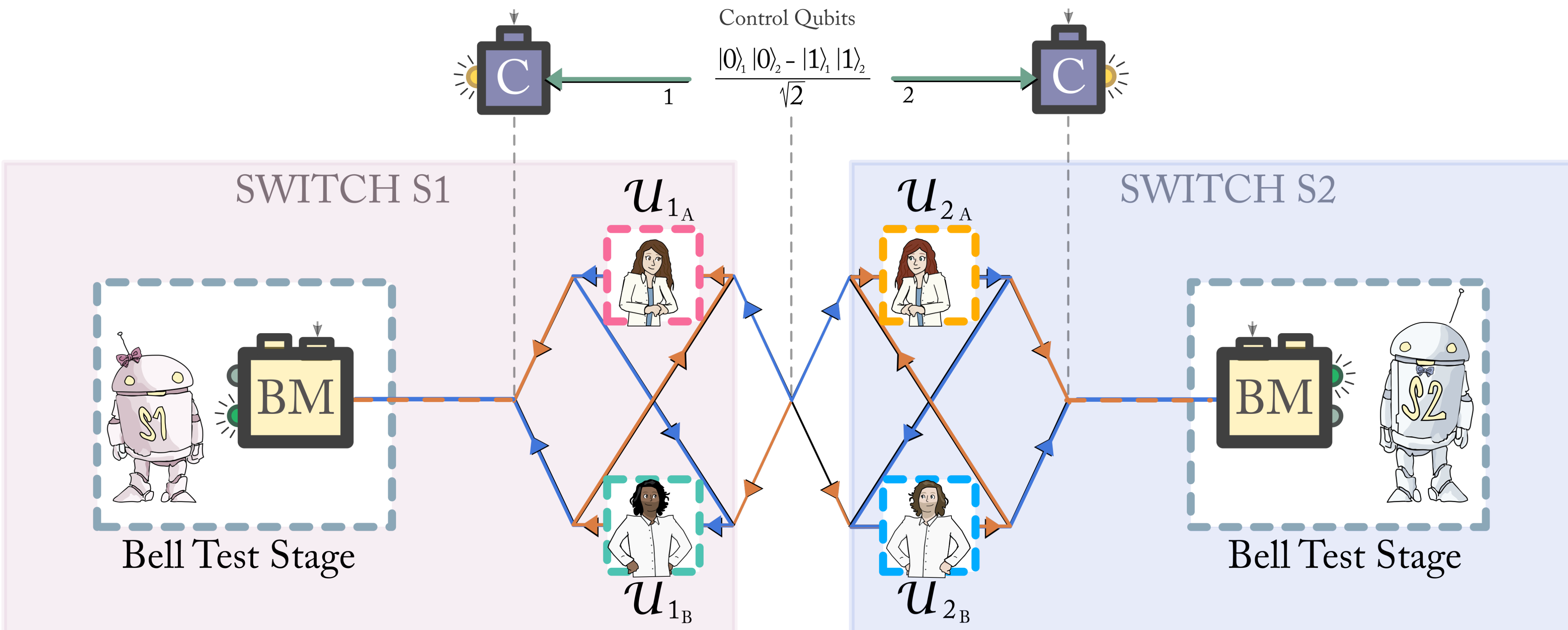


4) Choice of unitaries:

$$\mathcal{U}_{1_A} = \mathcal{U}_{2_A} = \sigma_z \qquad \mathcal{U}_{1_B} = \mathcal{U}_{2_B} = \frac{\mathcal{I} + i\sigma_x}{\sqrt{2}}$$

A NO-GO THEOREM FOR TEMPORAL ORDERS



⇒ Final state:

$$\frac{1}{\sqrt{2}}\left(|L\rangle_1^T|L\rangle_2^T - |R\rangle_1^T|R\rangle_2^T\right)$$

A NO-GO THEOREM FOR TEMPORAL ORDERS

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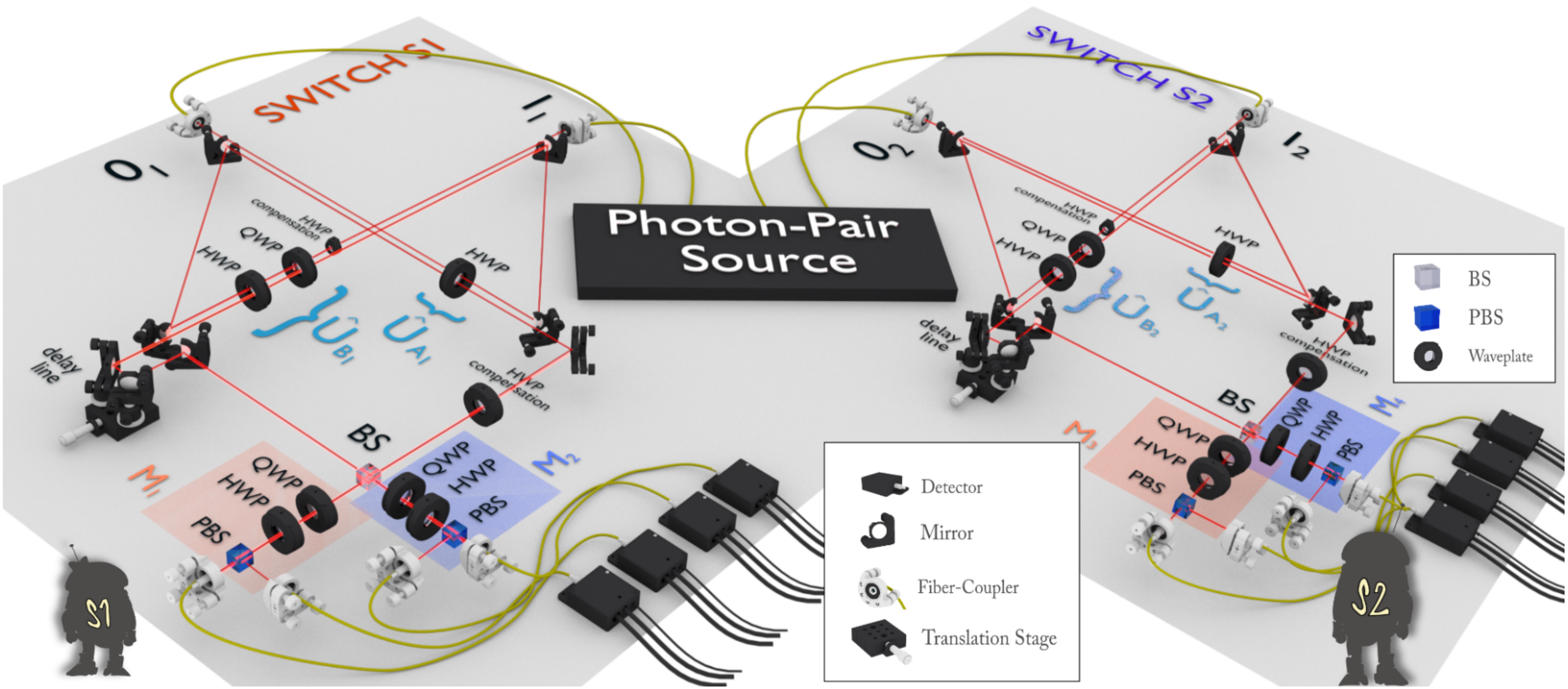
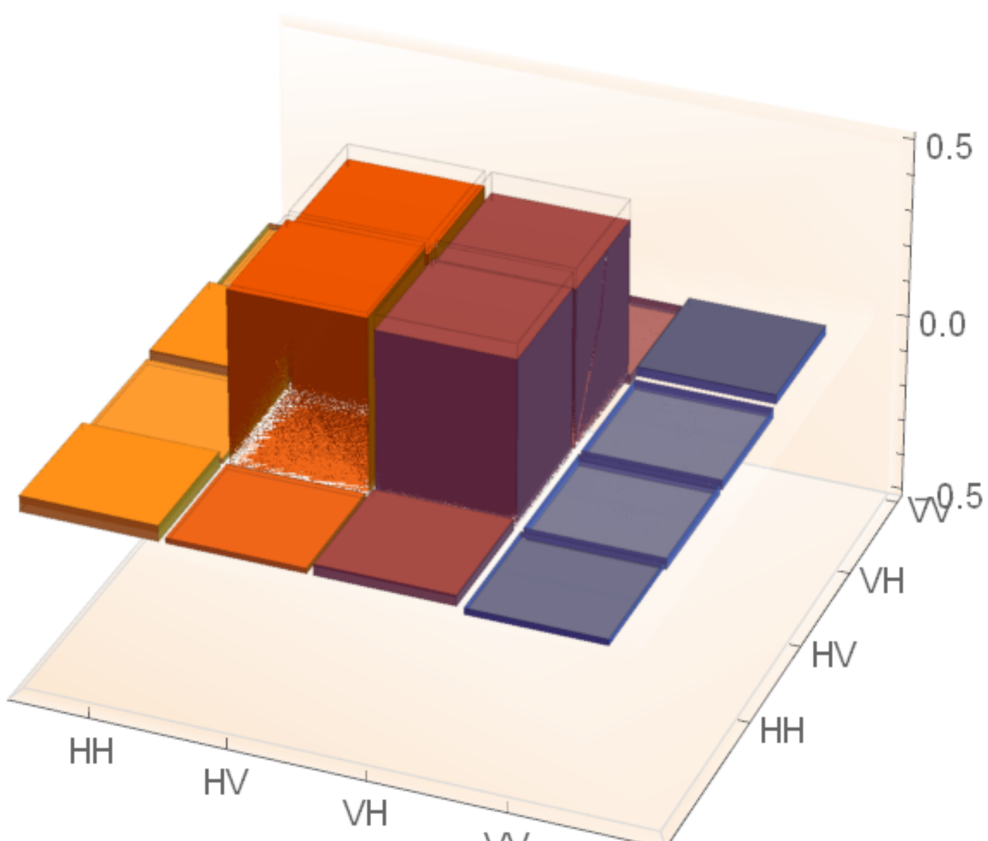
EXPERIMENTAL REALISATION AND RESULTS

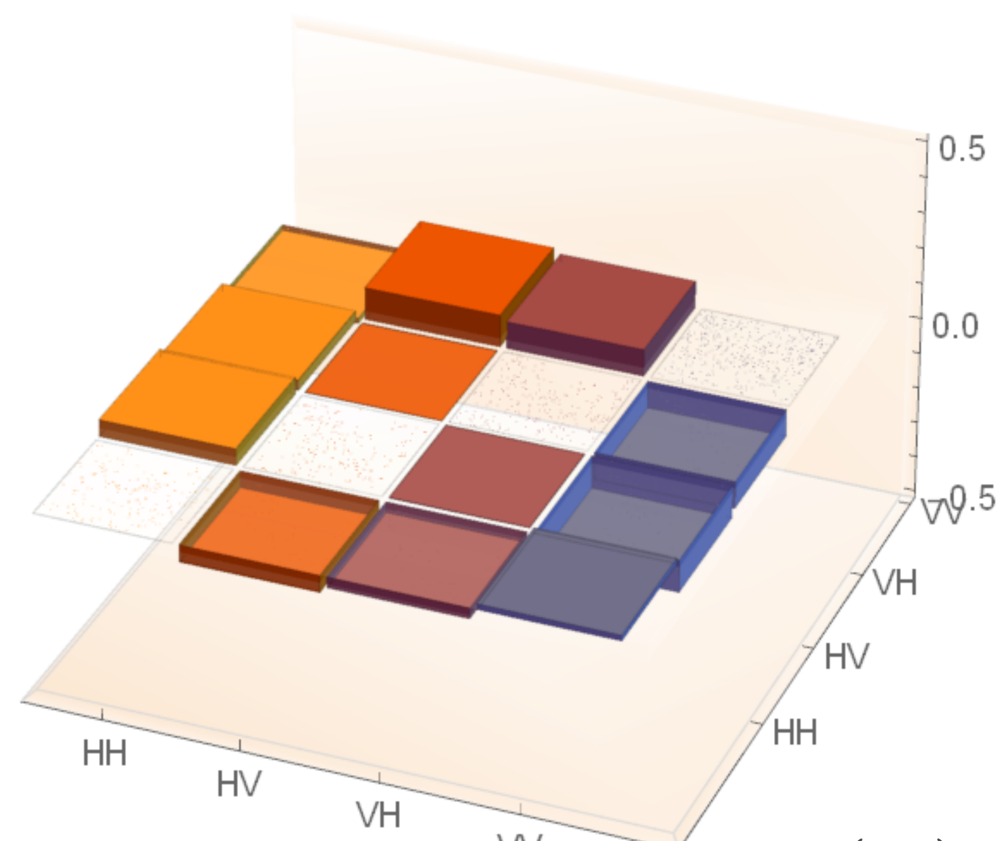


EXPERIMENTAL REALISATION AND RESULTS

Step 1: violation of the no-go theorem.



$\Re(\rho)$



$\Im(\rho)$

$$S_{\text{target}} = 2.55 \pm 0.08$$

$\text{Fidelity} = 0.922 \pm 0.005$ $\text{Concurrence} = 0.95 \pm 0.01$

EXPERIMENTAL REALISATION AND RESULTS

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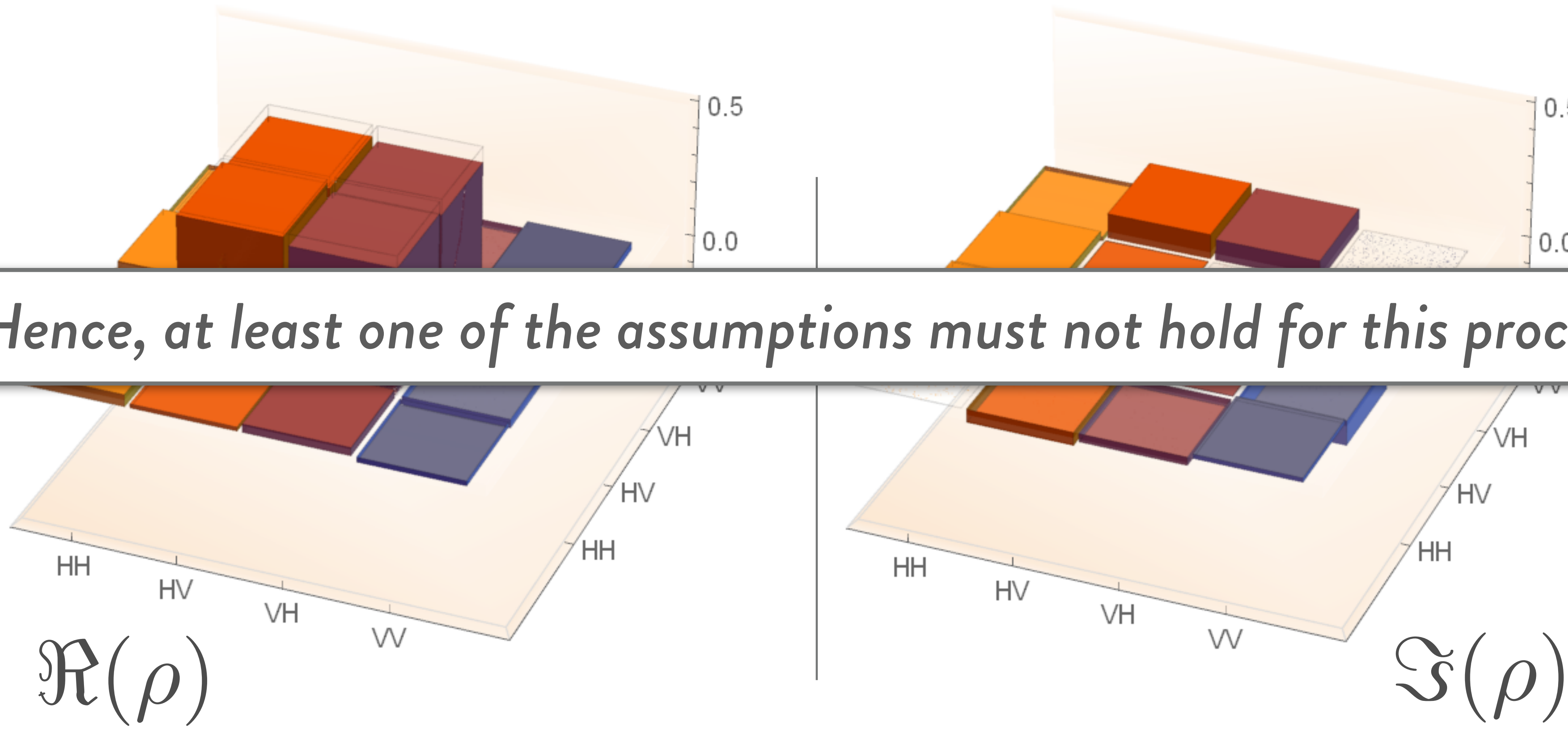

Hence, at least one of the assumptions must not hold for this process.

$$S_{\text{target}} = 2.55 \pm 0.08$$

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EXPERIMENTAL REALISATION AND RESULTS

Step 2: verification of assumption **1**.

Probabilities for measurement outcomes as measured on reduced states of target system of S1 and S2:

$$p(o_1, o_2 | m_1, m_2, \omega^T_{1,2}) = p(o_1 | m_1, \omega^T_1) \cdot p(o_2 | m_2, \omega^T_2)$$

The probability for joint outcomes on the composite system of the two targets $\omega^T_{1,2}$ in the initial state is *factorisable.*

EXPERIMENTAL REALISATION AND RESULTS

Step 2: verification of assumption **1**.

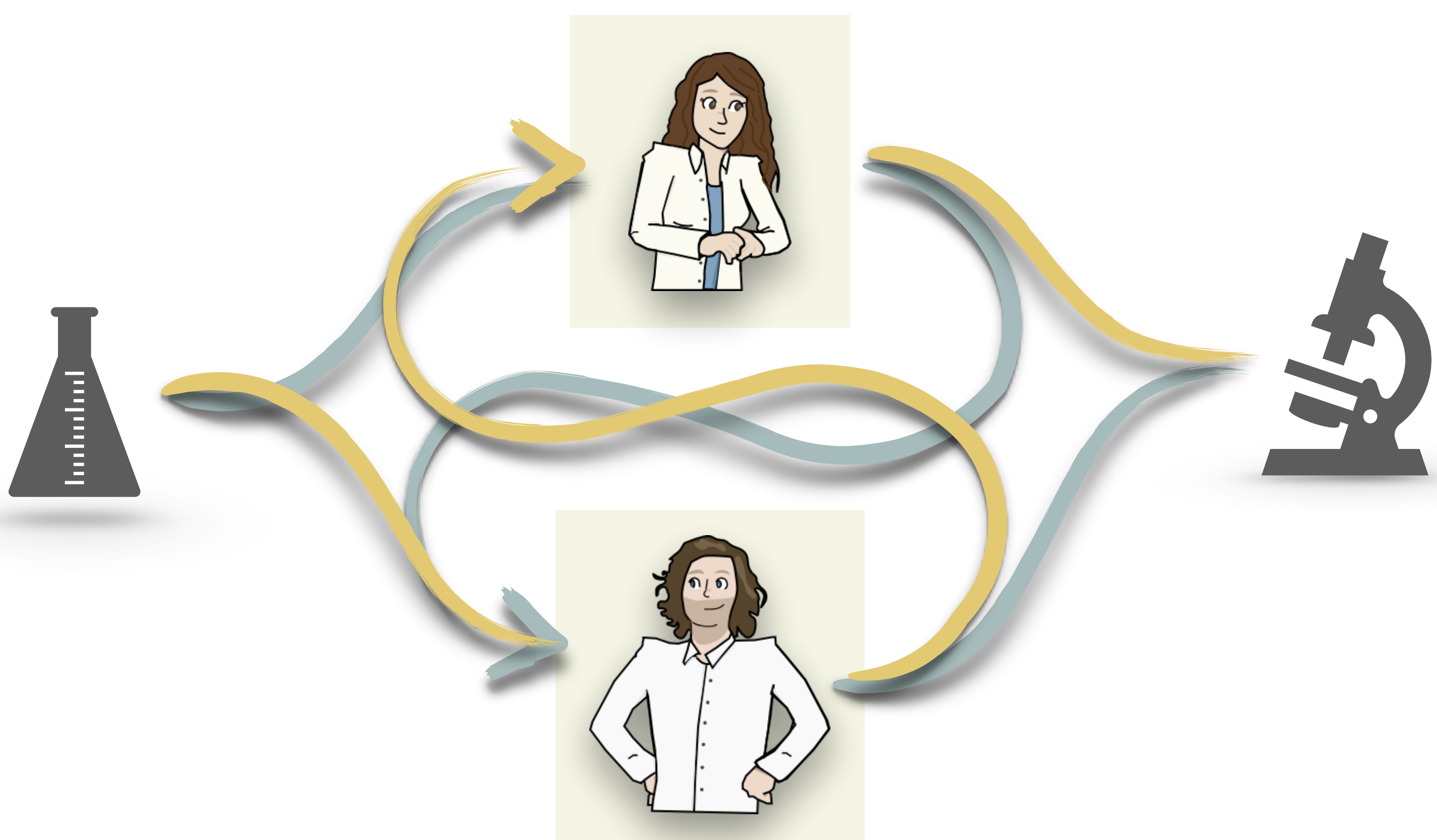
The distance between these two sets of probabilities is $3.2 \cdot 10^{-2}$

Measur. Basis	$p(\omega^T_{1,2})$	$p(\omega^T_{1,2^\perp})$	$p(\omega^T_{1^\perp,2})$	$p(\omega^T_{1^\perp,2^\perp})$	$p(\omega^T_1) \cdot p(\omega^T_2)$	$p(\omega^T_1) \cdot p(\omega^T_{2^\perp})$	$p(\omega^T_{1^\perp}) \cdot p(\omega^T_2)$	$p(\omega^T_{1^\perp}) \cdot p(\omega^T_{2^\perp})$
H, H	$\boxed{0.99}$	0.03	0.00	0.00	$\boxed{1.00}$	0.03	0.00	0.00
H, V	0.01	0.97	0.00	0.00	0.01	0.95	0.00	0.00
H, A	0.56	0.41	0.00	0.00	0.55	0.39	0.00	0.00
H, D	0.43	0.59	0.00	0.00	0.45	0.61	0.00	0.00
H, R	0.39	0.62	0.00	0.00	0.40	0.63	0.00	0.00
H, L	0.60	0.38	0.00	0.00	0.59	0.37	0.00	0.00
V, H	0.00	0.00	0.97	0.04	0.00	0.00	0.98	0.04
V, V	0.00	0.00	0.03	0.96	0.00	0.00	0.03	0.95
V, A	0.00	0.00	0.63	0.39	0.00	0.00	0.64	0.40
V, D	0.00	0.00	0.37	0.61	0.00	0.00	0.37	0.59
V, R	0.00	0.00	0.34	0.63	0.00	0.00	0.33	0.62
V, L	0.00	0.00	0.66	0.37	0.00	0.00	0.68	0.38
A, H	0.33	0.01	0.48	0.02	0.27	0.01	0.40	0.01

EXPERIMENTAL REALISATION AND RESULTS

Step 3: verification of assumptions **2**. and **3**.

Either assumption **2.** does not hold, assumption **3.** does not hold, or both assumptions are invalid.



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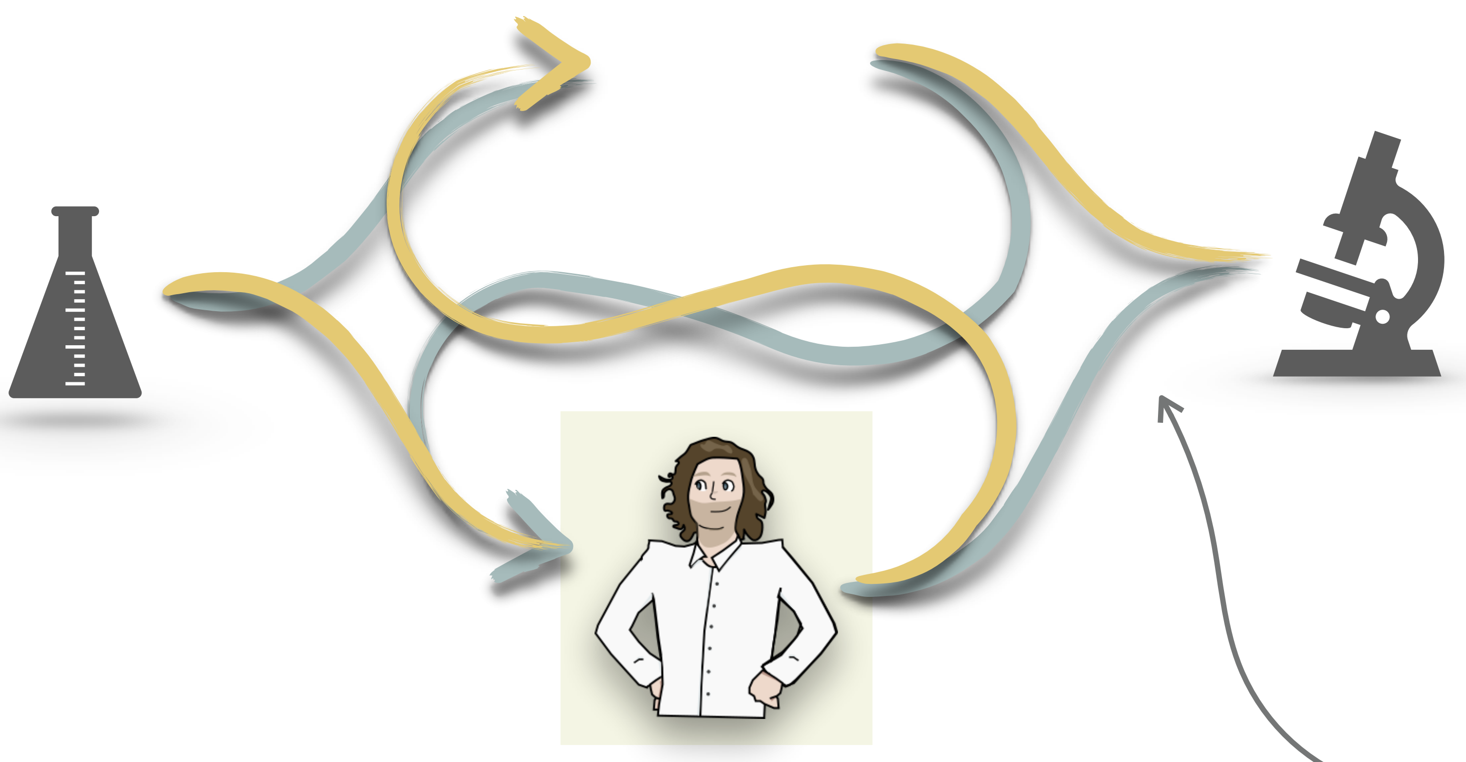

no coupling between control and target system

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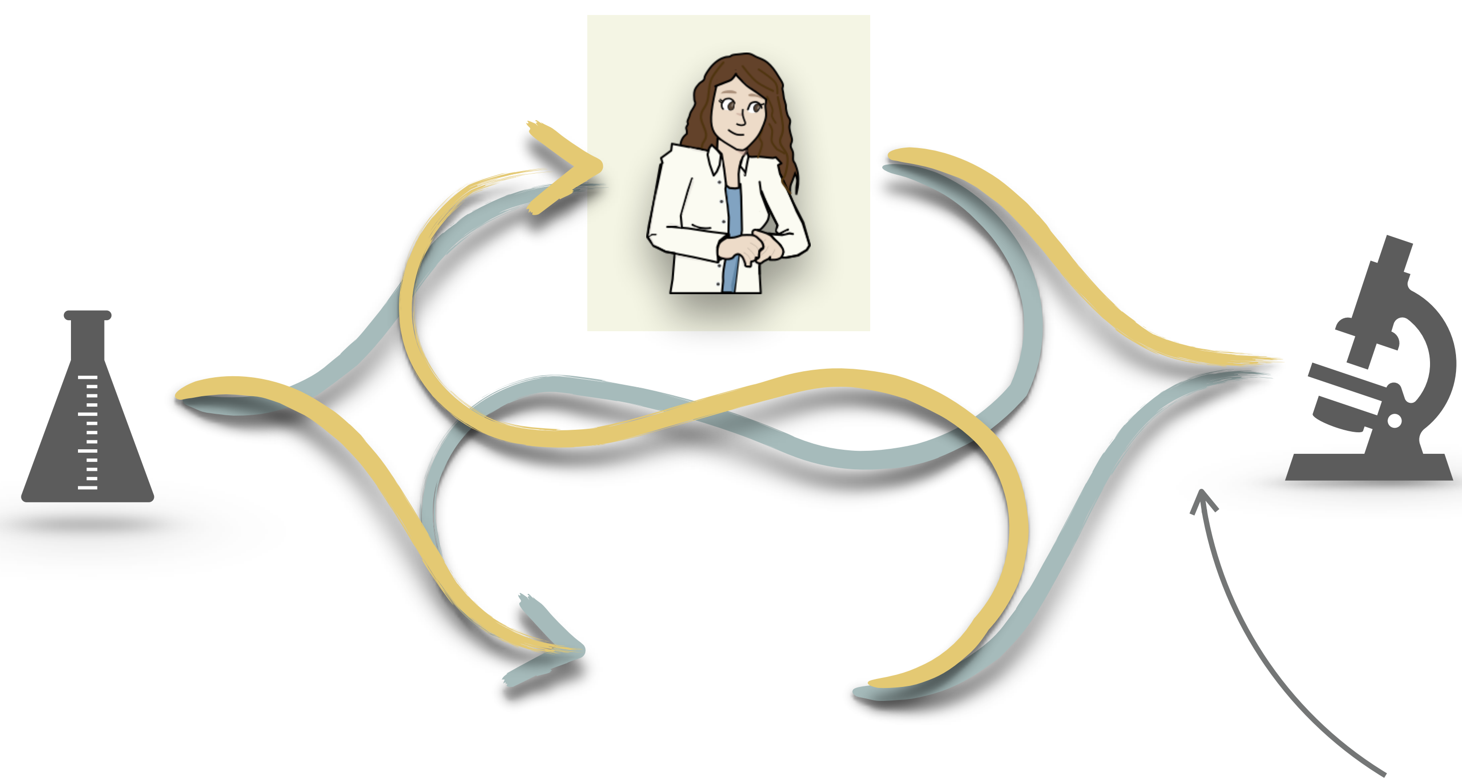

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EXPERIMENTAL REALISATION AND RESULTS

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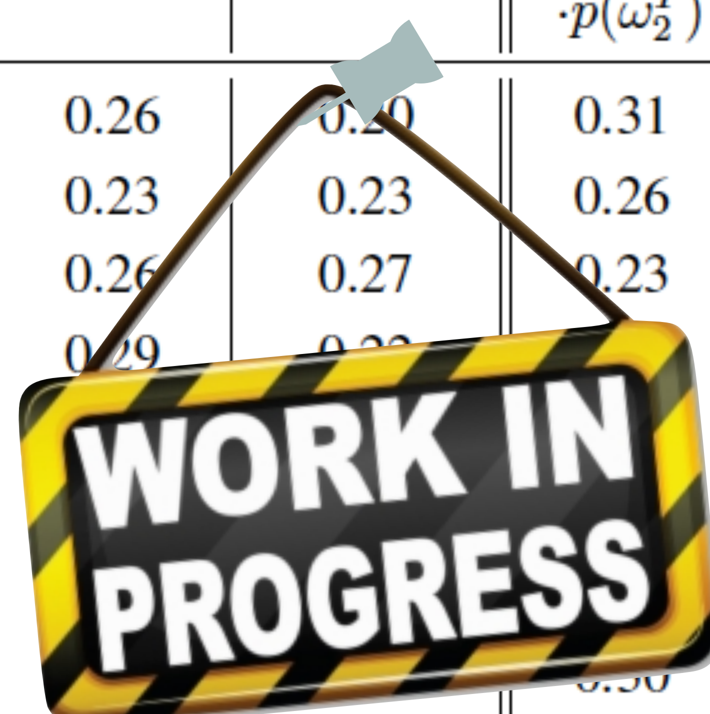
Measur. Basis	$p(\omega^T_{1,2})$	$p(\omega^T_{1,2^\perp})$	$p(\omega^T_{1^\perp,2})$	$p(\omega^T_{1^\perp,2^\perp})$	$p(\omega^T_1) \cdot p(\omega^T_2)$	$p(\omega^T_1) \cdot p(\omega^T_{2^\perp})$	$p(\omega^T_{1^\perp}) \cdot p(\omega^T_2)$	$p(\omega^T_{1^\perp}) \cdot p(\omega^T_{2^\perp})$
H, H	0.31	0.23	0.26	[illegible]	0.31	0.23	0.26	0.20
H, V	0.27	0.27	0.23	0.23	0.26	0.27	0.23	0.23
V, V	0.23	0.24	0.26	0.27	0.23	0.24	0.26	0.27
V, H	0.28	0.21	0.29	[illegible]	[illegible]	0.21	0.29	0.22
R, H	0.02	0.01	[illegible]	[illegible]	[illegible]	0.01	0.54	0.43
R, V	0.01	0.01	[illegible]	[illegible]	[illegible]	0.01	0.49	0.49
D, V	0.26	0.27	[illegible]	[illegible]	[illegible]	0.27	0.23	0.24
D, H	0.31	0.23	[illegible]	[illegible]	[illegible]	0.24	0.26	0.20
D, R	0.02	0.50	0.02	0.46	0.02	0.51	0.02	0.45
D, D	0.28	0.24	0.26	0.21	0.29	0.24	0.26	0.22
R, D	0.01	0.01	0.54	0.44	0.01	0.01	0.54	0.44



EXPERIMENTAL REALISATION AND RESULTS

Step 3: verification of assumptions **2**. and **3**.

This will show that, whenever assumption **3**. is verified, so is assumption **2**.

Because $A \Rightarrow B \quad \equiv \quad \text{not } B \Rightarrow \text{not } A$ the only two possible solutions are that

(1) assumption **2**. is wrong, and therefore **3**. is wrong.

(2) assumption **3**. is false, independently of the status of **2**.

In either case, assumption **3**. is invalid, and therefore ***the scenarios in which the operations were applied in a well-defined order are to be excluded***.

EXPERIMENTAL REALISATION AND RESULTS

- ✓ First demonstration of a process that is incompatible with a large class of theories which are *local* and have a *definite temporal order.*
- ✓ Entangled quantum SWITCH.

WHAT NEXT?

- Scaling up the number of parties.
- Perform more complex operations on the control qubit.

THANK YOU FOR YOUR ATTENTION

REFERENCES

[1] M. Zych, *et al.*, Nat. Comm. **10**, 3772 (2019).

[2] O. Oreshkov, F. Costa, Č. Brukner, Nat. Commun. **3**, 1092 (2012).

[3] G. Chiribella, *et al.*, Phys. Rev. A **88**, 022318 (2013).

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[8] G. Chiribella, Phys. Rev. A **86**, 040301 (2012).

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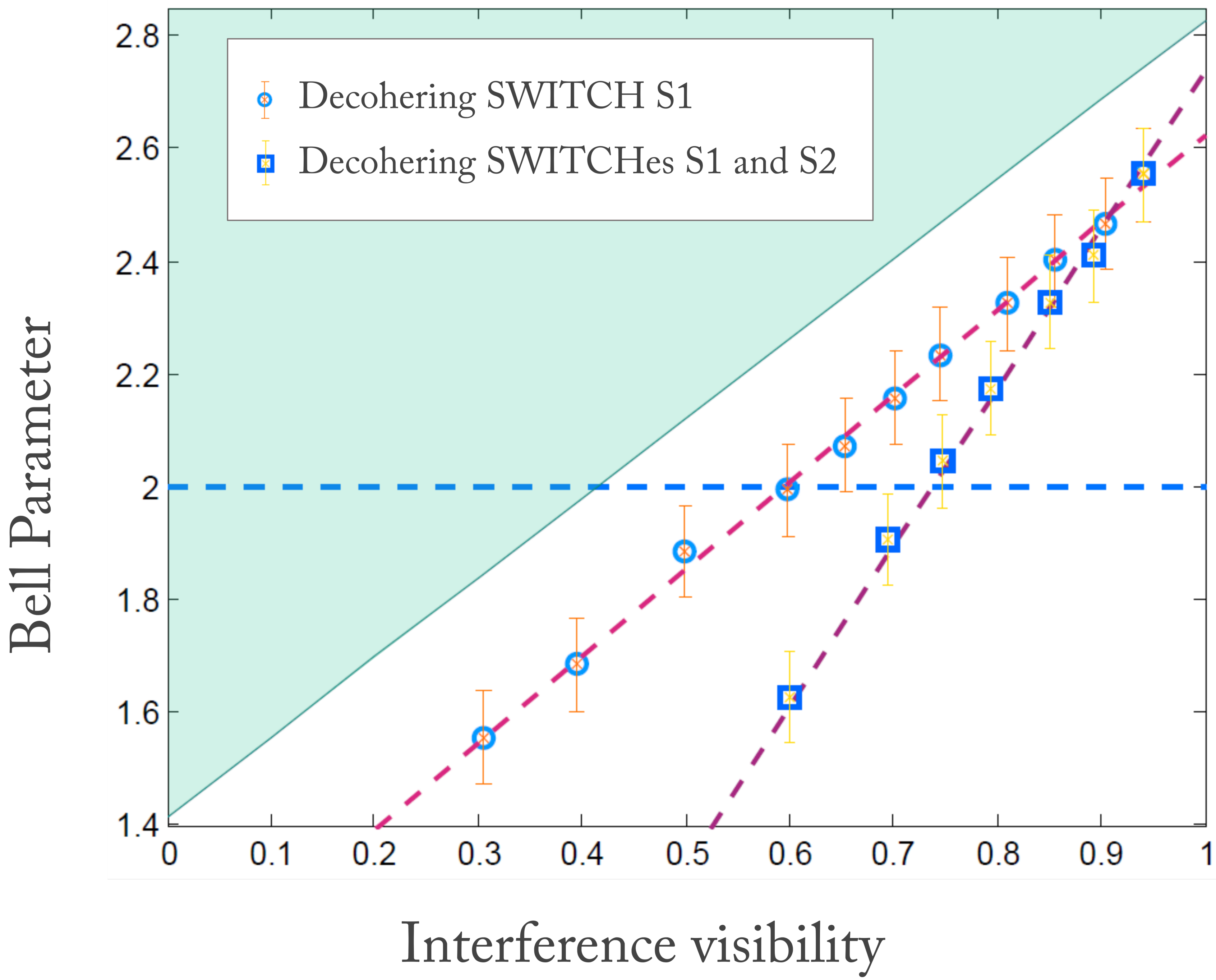
[10] M. Araújo, *et al.*, New Journal of Physics **17**, 102001 (2015).

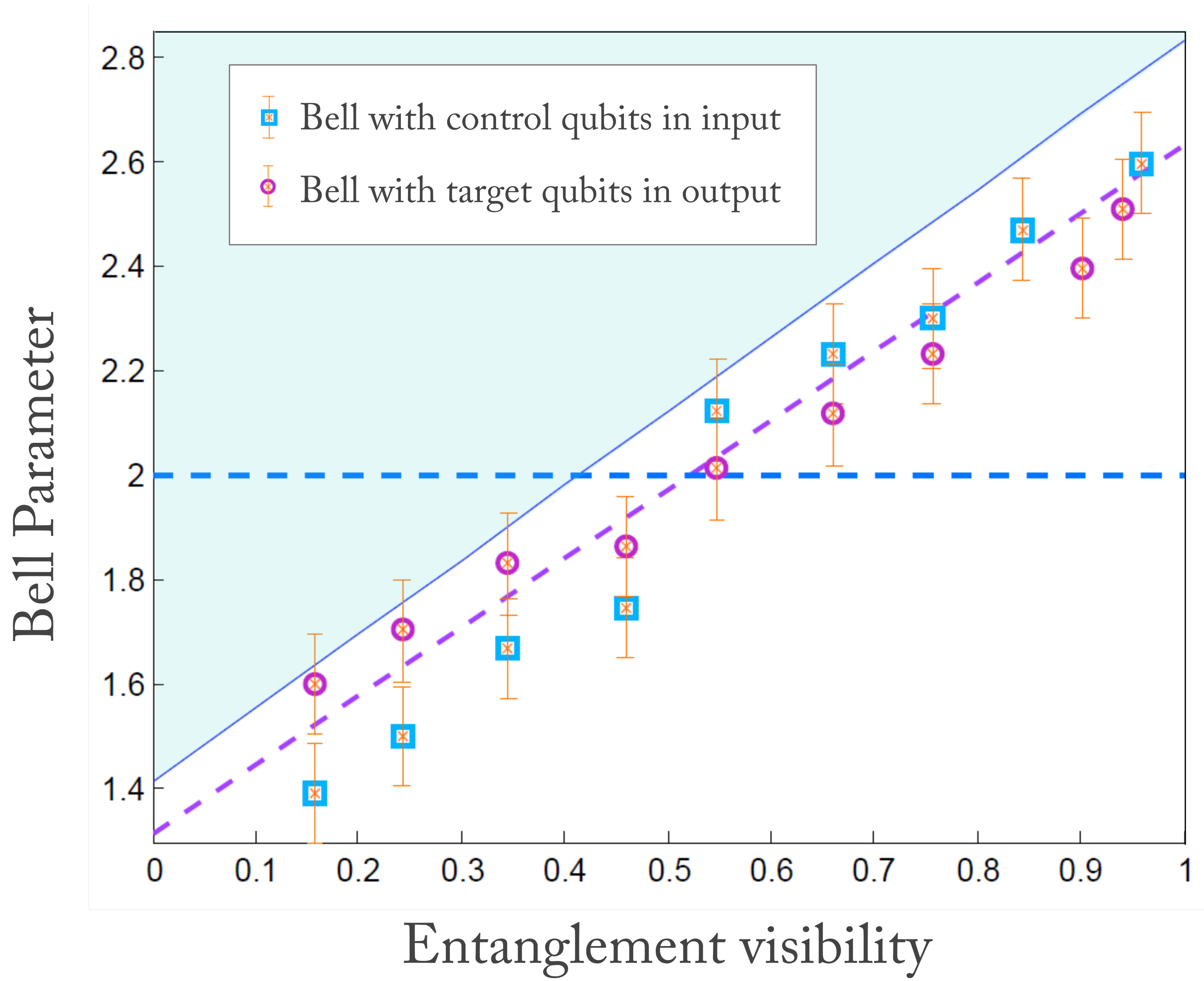
EXPERIMENTAL REALISATION AND RESULTS

Insertion of noise - rephrasing the output control system



EXPERIMENTAL REALISATION AND RESULTS

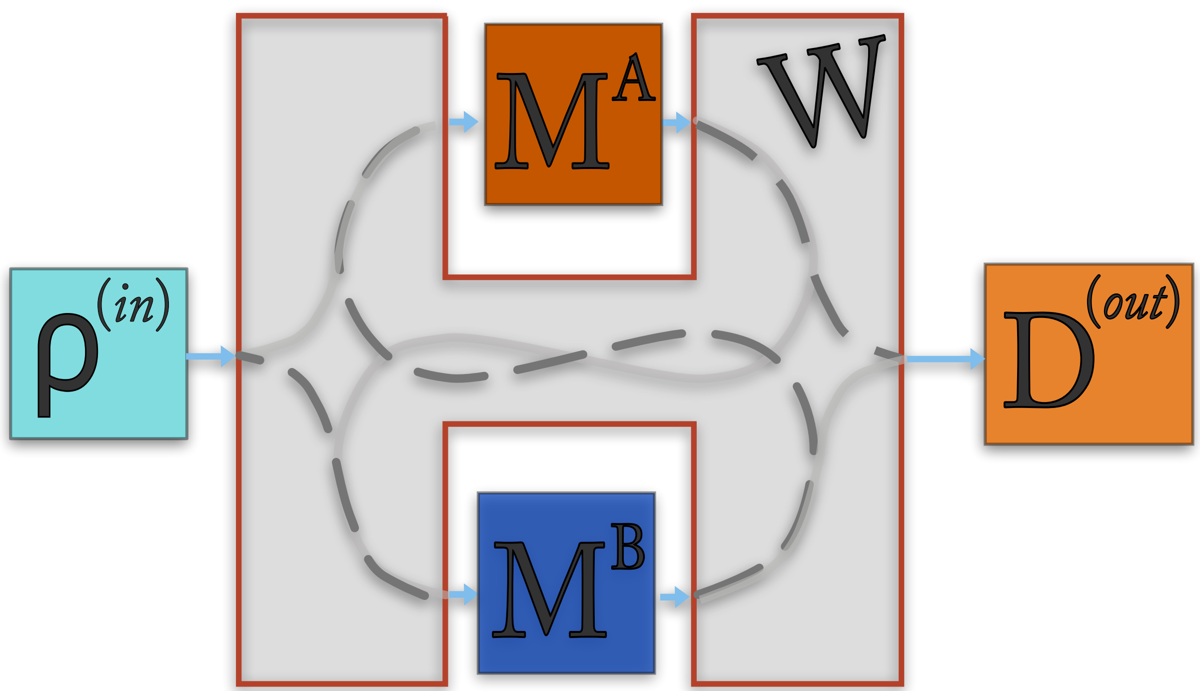
Insertion of noise - in the entanglement of the input control system



genuinely indefinite causal structure:

a quantum process is called causally separable if it can be decomposed as a convex combination of causally ordered processes, otherwise it is causally non-separable.

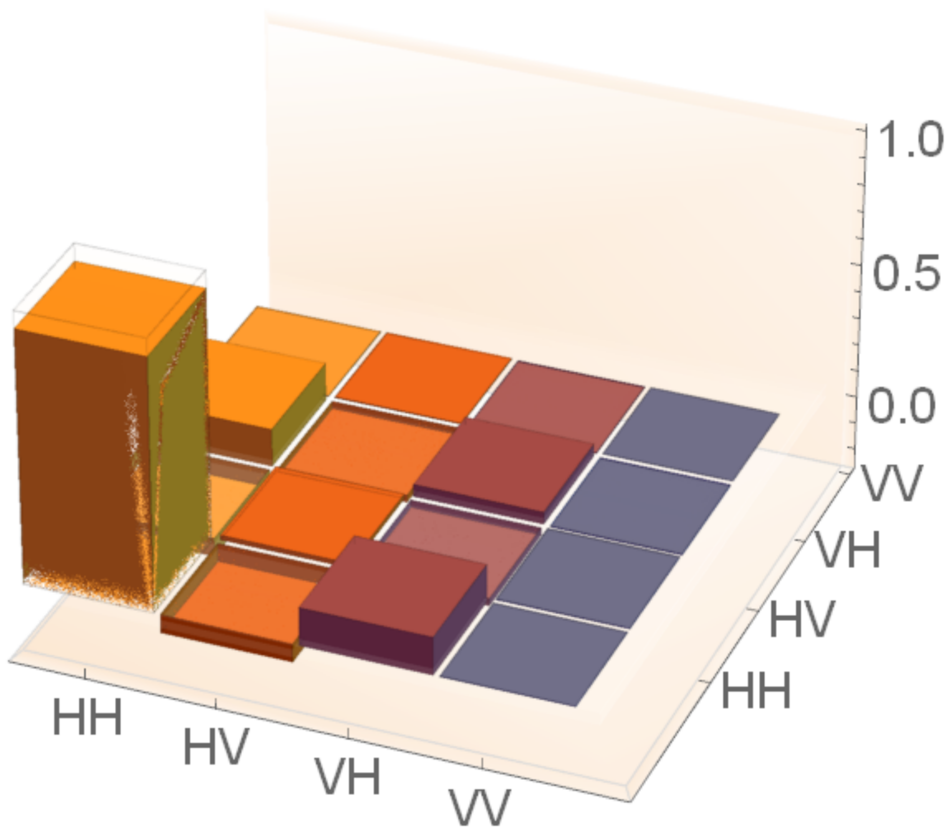
Process Matrix:



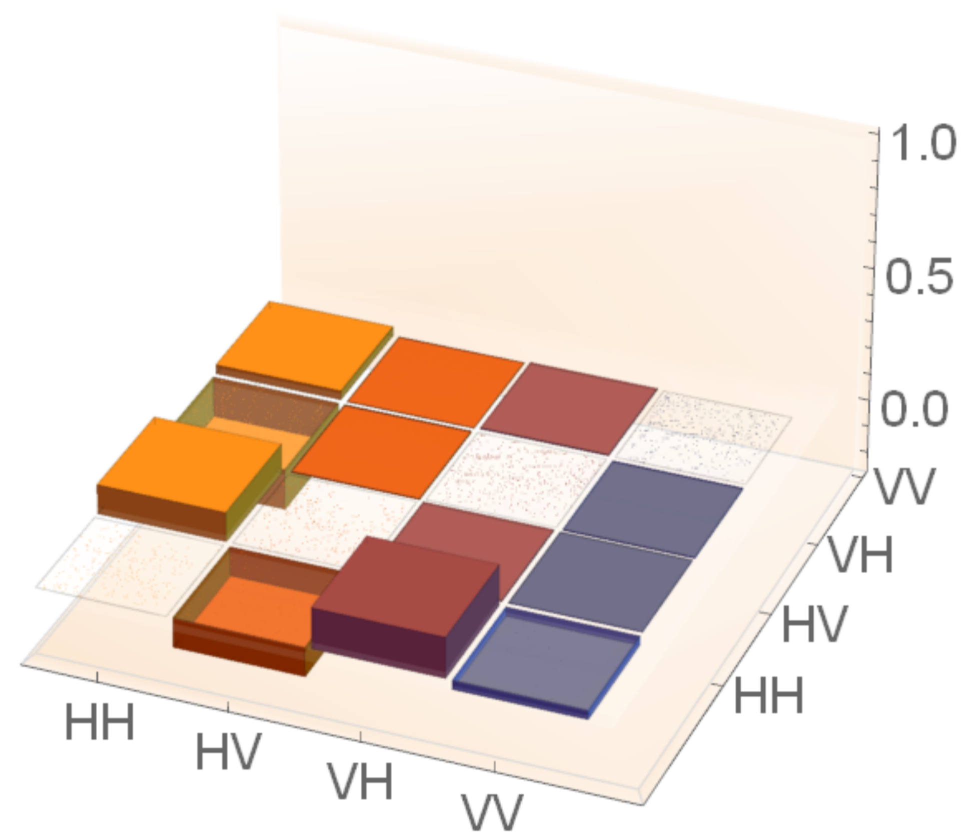
$$p(a,b,d|x,y,z) = \mathrm{Tr}\left[\left(\rho_z^{(\mathrm{in})} \otimes M_{a,x}^A \otimes M_{b,y}^B \otimes D_d^{(\mathrm{out})}\right) \cdot W\right]$$

EXPERIMENTAL REALISATION AND RESULTS

Step 2: verification of assumption **1**.



$\Re(\rho)$



$\Im(\rho)$

$\text{Fidelity} = 0.935 \pm 0.004$ $\quad$ $\text{Concurrence} = 0.001 \pm 0.010$